The cooling of a heat-generating board inside a parallel-plate channel

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This paper addresses the fundamental question of how to position a heat-generating board inside a parallel-plate channel, where it is cooled by forced convection. It is shown that when the board substrate is a relatively good thermal conductor, the best board position is near one of the channel walls, and the worst position is in the middle of the channel. The best and worst positions switch places when the board substrate is a relatively poor conductor. The optimal spacing between a heat-generating surface (uniform temperature, or uniform heat flux) and the insulated wall that completes a parallel-plate channel is reported. Finally, it is shown under what conditions it is advantageous to divide a heat-generating board into two or more equidistant boards inside the same channel, when the total rate of heat generation of all the boards and the channel spacing are fixed.

Keywords: electronic packages; internal forced convection; fins

1. Introduction

The objective of this paper is to address the fundamental thermal design problem of how to position a heat-generating board inside a parallel-plate channel with forced convection cooling, so that the board temperature is minimum. This question will be answered in a sequence of steps that begins with the simplest: (1) what is the optimal position of a highly conducting board inside a parallel-plate channel (section 2), (2) what is the optimal position of a board with finite transversal thermal conductance (section 3), (3) what is the optimal spacing of a channel with the heated plate attached to one of its walls (sections 4 and 5), and (4) under what circumstances should the single board be replaced with two or more equidistant boards that generate the same (total) rate of Joule heating (section 6).

The fundamental thermal design questions formulated in this paper were motivated by a real problem encountered in the electronics industry. In certain types of electronic packages, the single-phase coolant flows through a set of two-dimensional (2-D) parallel channels formed by a row of printed circuit boards plugged into a mother board. Each board may be surrounded by a metal or metal-coated plastic case whose function is to shield the electronic circuitry from external electromagnetic noise. It is important to know the optimal geometry of each "cassette" (i.e., the board and its parallel-plate casing) so that the board operating temperature is minimum. To optimize the geometry of the cassette means to find not only the optimal position for the board inside the channel, but also the optimal slenderness of the cassette itself (i.e., the spacing of the channel in which the heat-generating board is encased).

Recent review articles (Incropera 1988; Peterson and Ortega 1990) show that the problems solved in this paper have not been addressed in the electronic cooling literature. On the other hand, the fundamentals of heat and fluid flow through multiple channels have received considerable attention in the heat-exchanger literature (Rohsenow *et al.* 1985; Shah and London 1978).

2. The optimal position of a heated plate inside a parallel-plate channel

Consider first the challenge of cooling in the most effective way a heated plate of length L, by positioning it in a stream of coolant that flows through an insulated parallel-plate channel of the same length. The channel spacing D is fixed. The geometry sketched in Figure 1 is 2-D, since the blade and the channel are sufficiently wide (width $= W$) in the direction perpendicular to the figure, $W > L$.

We assume that the pressure difference across the channel, ΔP, is fixed because the flow is driven by a fan with diameter considerably greater than the channel spacing D. In an actual application, the fan would blow air through a stack of ten or more cassettes of profile $L \times D$: only one such cassette is presented in Figure 1.

The heated plate is a simplified model for a printed circuit board, or a blade-shaped electric resistance heater. The total rate of heat transfer q from the heated plate to the fluid, through *both* sides of the plate, is fixed by the electric circuit design. The plate thickness is negligible with respect to the channel spacing D. The only degree of freedom in choosing the best cooling arrangement is the position of the heated plate inside the channel. This position is pinpointed by the subchannel

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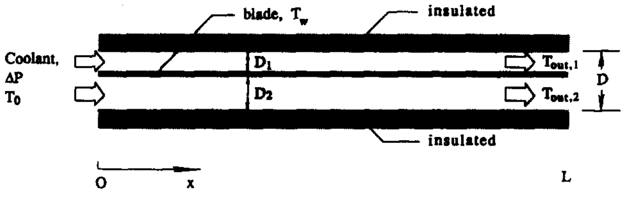

Figure 1 Heated plate cooled by a stream ducted through an insulated parallel-plate channel

spacings above and below the heated plate, D_1 and D_2, such that $D_1 + D_2 = D$.

To illustrate the solution method in the simplest possible way, in this section we make the following assumptions:

(a) the heated plate is isothermal at T_w;
(b) the flow is fully developed and laminar on both sides of the plate;
(c) the NTU on either side of the plate is sufficiently greater than 1 so that $(T_{out,1,2} - T_0)/(T_w - T_0) \cong 1$; and
(d) the surfaces of the plate and the channel walls are smooth.

As shown in Figure 1, $T_{out,1}$ and $T_{out,2}$ are the outlet bulk temperatures, above and below the heated plate. The objective is to determine the best configuration (D_1/D) so that the thermal conductance $q/(T_w - T_0)$ is the greatest.

If we label q_1 and q_2 the heat transfer rates through the upper side and the lower side of the heated plate, respectively, then the approximation (c) above permits us to write

$$q_1 = \dot{m}_1 c_P(T_w - T_0) \tag{1}$$

$$q_2 = \dot{m}_2 c_P(T_w - T_0) \tag{2}$$

where, for fully developed laminar flow (Incropera 1988),

$$\dot{m}_1 = \frac{\rho W}{12\mu}\frac{\Delta P}{L} D_1^3 \tag{3}$$

$$\dot{m}_2 = \frac{\rho W}{12\mu}\frac{\Delta P}{L} D_2^3 \tag{4}$$

Next, we write y and $(1 - y)$ for the dimensionless spacings of the upper and lower subchannels,

$$D_1 = yD, \qquad D_2 = (1 - y)D \tag{5}$$

and calculate the total heat transfer rate $q = q_1 + q_2$, by using Equations 1 to 4. The result is

$$\frac{q}{T_w - T_0}\,\frac{12\mu L}{\rho\, c_p W D^3 \Delta P} = y^3 + (1 - y)^3 \tag{6}$$

Figure 2 shows that the highest value of the y function on the right-hand side is 1, and that it occurs when $y = 0$, or $y = 1$.

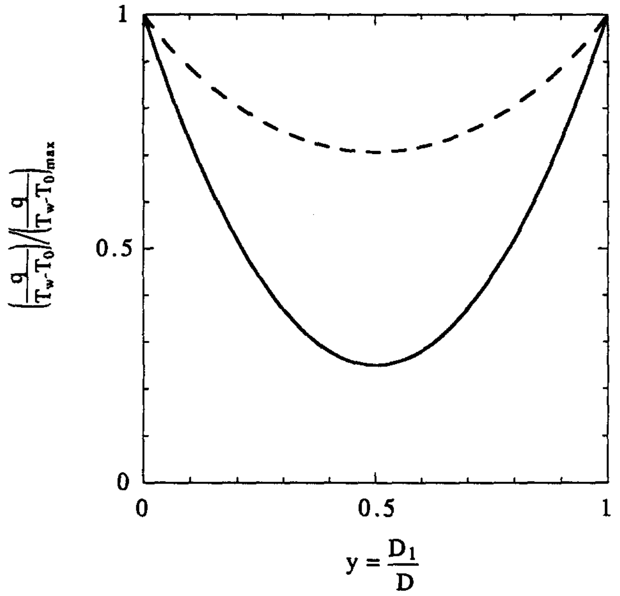

Figure 2 The effect of the position of the heated plate on the overall thermal conductance: ——— laminar, fully developed; ---- turbulent, fully rough, fully developed

Notation

a, b	Parameters, Equation 19
A_w	Heat transfer area, $L \times W$
B	Dimensionless board transversal thermal conductance, Equation 17
c_P	Specific heat at constant pressure
D	Channel spacing
D_h	Hydraulic diameter, $2D$
h	Heat transfer coefficient
k	Thermal conductivity of fluid
k_w	Thermal conductivity of board
L	Length
$\dot{m}$	Mass flowrate
n	Number of boards, Figure 8
NTU	Number of heat transfer units, Equation 23
Nu	Nusselt number, hD_h/k
p	Parameter, Equation 19
q	Total heat transfer rate
q''	Uniform heat flux
$\bar{q}''$	Average heat flux
t	Board thickness
T_h	Highest temperature, Figure 4
T_{out}	Stream outlet temperature
T_w	Uniform temperature
T_0	Stream inlet temperature
U	Mean velocity
W	Width, perpendicular to Figures 1, 3, and 4
y	Dimensionless location of inserted plate, Equation 5
y_{max}	Worst location, Figure 4
y_{min}	Best location, Figure 4

Greek symbols

α	Thermal diffusivity
δ	Dimensionless spacing, Equation 12
ΔP	Pressure difference
$\theta_{1,2}$	Dimensionless trailing-edge temperatures, Equation 14
θ_{max}	Uppermost temperature ceiling, Figure 5
θ_{min}	Lowest temperature ceiling, Figure 5
μ	Viscosity
ν	Kinematic viscosity
Π	Dimensionless pressure difference, Equation 24
ρ	Fluid density

Subscripts

h	Highest temperature
max	Maximum
opt	Optimal
1	The upper subchannel, Figure 1
2	The lower subchannel, Figure 1

The minimum value ($\frac{1}{4}$) occurs when $y = \frac{1}{2}$. We reach the somewhat unexpected conclusion that, from a cooling standpoint, the centerplane ($y = \frac{1}{2}$) is the worst position that the heated plate can have. The best arrangement is the one where the plate is attached to one of the insulated walls of the channel, even though in that case the entire heat transfer rate q must leave the plate through only one of its side surfaces. When the blade is attached to one of the walls, the thermal conductance $q/(T_w - T_0)$ is four times greater than when the blade is positioned in the center of the channel.

The conclusion that the worst cooling position is $y = \frac{1}{2}$ remains valid even when some of the simplifying assumptions (a)–(d) are relaxed. For example, let us discard assumptions (b) and (d) together, and assume instead that the flow is turbulent fully developed, and the board surfaces are very rough. This is a good model for cassettes with L/D ratios much greater than 10, so that the entrance region is relatively small, and with boards densely covered with chips and circuitry that rise as large scale three-dimensional (3-D) asperities above the surface. Under these circumstances (i.e., the "fully rough" limit of turbulent duct flow), the friction factor is practically independent of the Reynolds number (Incropera and DeWitt 1990), and Equations 3 and 4 are replaced by

$$\dot{m}_1 = \left(\frac{\rho\,\Delta P}{f_1 L}\right)^{1/2} W D_1^{3/2} \qquad (7)$$

$$\dot{m}_2 = \left(\frac{\rho\,\Delta P}{f_2 L}\right)^{1/2} W D_2^{3/2} \qquad (8)$$

The friction factor constants f_1 and f_2 depend on the dimensions of the roughness elements (assumed the same for both board surfaces) and on the respective subchannel spacings (D_1, D_2). When the board is placed "in the stream," i.e., at y values comparable with $\frac{1}{2}$, the spacings D_1 and D_2 are also comparable and, as a first approximation, f_1 and f_2 may be taken as equal to the same constant f. This is a conservative approximation to which we shall return in the paragraph after next. In the end, Equation 6 is replaced by

$$\frac{q}{T_w - T_0}\left(\frac{fL}{\rho\,\Delta P}\right)^{1/2}\frac{1}{c_P W D^{3/2}} = y^{3/2} + (1 - y)^{3/2} \qquad (9)$$

This result shows that the overall thermal conductance $q/(T_w - T_0)$ is once again minimum if the board is placed in the middle of the channel. The right-hand side of Equation 9 is plotted as a dash curve in Figure 2, and is valid in the vicinity of $y = \frac{1}{2}$. The thermal conductance minimum is not as sharp as for the fully developed laminar flow, suggesting that the optimal positioning of the board is not as critical in the fully rough limit.

If one is to repeat the analysis and take into account the difference between f_1 and f_2 as the board is positioned close to one of the walls, one would obtain a curve that falls under the dash curve in Figure 2. The reason is that in the fully rough regime, the friction factor decreases weakly as the channel spacing increases (see, for example, Bejan 1984). This weak dependence acts in the direction of increasing (slightly) the $\frac{3}{2}$ exponent that appears in Equations 7 to 9. The new curve would fall under the dash curve (exponent $= \frac{3}{2}$) and above the solid curve (exponent $= 3$; see Equation 6).

3. Heat-generating board with finite thermal conductance in the transversal direction

The conclusion reached in the preceding section and in Figure 2 (namely, $y_{\text{opt}} = 0{,}1$) is valid when the board thermal conductivity is so high that the temperature T_w can be regarded as uniform. Only such a board is capable of releasing its entire rate of heat generation to a single subchannel ($D_1 = D$, or $D_2 = D$) when the board is positioned near one of the walls.

Consider now the more realistic model in which the board of Figure 1 (the substrate of an electronic circuit board) has a finite thermal conductivity k_w and thickness t. The thickness continues to be negligible with respect to D. The two surfaces of the board are loaded equally and uniformly with electronics: the constant heat generation rate per unit board surface is q''. It is important to note, however, that the heat fluxes removed by the two streams generally are not equal, because of the conduction heat transfer across the board.

The temperatures of the two board surfaces (T_1, T_2) increase in the downstream direction, and reach their highest levels at the trailing edge, $x = L$. The objective is to minimize the larger of these two trailing-edge temperatures, by choosing the optimal board position y.

We obtain the temperature distributions $T_1(x)$ and $T_2(x)$ by making the simplifying assumption that the temperature increase along each surface (for example, $T_1(L) - T_1(0)$) is considerably greater than the local temperature difference between the surface and the corresponding stream. This assumption becomes better as the $D \times L$ channel becomes more slender. It means that the local temperature of each stream ($\dot{m}_1, \dot{m}_2$) is fairly close to the temperature of the neighboring spot on the board surface bathed by the stream. Note that this assumption is the equivalent of assumption (c) in the preceding section. Based on it, we can write the first law for a dx slice of the D_1 and D_2 subchannels in the following way:

$$\dot{m}_1 c_P\, dT_1 = q_1'' W\, dx \qquad (10)$$

$$\dot{m}_2 c_P\, dT_2 = q_2'' W\, dx \qquad (11)$$

The mass flowrates $\dot{m}_1$ and $\dot{m}_2$ are given by Equations 3 and 4, as each subchannel stream is assumed laminar and fully developed. Although the heat flux generated by the electronics mounted on each surface of the board is uniform, q'', the heat fluxes removed by the two streams are influenced by the conduction heat current across the board $(k_w/t)(T_2 - T_1)$,

$$q_1'' = q'' + \frac{k_w}{t}(T_2 - T_1) \qquad (12)$$

$$q_2'' = q'' - \frac{k_w}{t}(T_2 - T_1) \qquad (13)$$

In Equations 3 and 4 and 10–13 we have all we need for determining the surface temperatures $T_1(x)$ and $T_2(x)$ subject to the entrance condition $T_1(0) = T_2(0) = T_0$. This operation becomes clearer if we use the dimensionless variables $D_1/D = y$, $D_2/D = 1 - y$, $\xi = x/L$, and

$$\theta_1 = (T_1 - T_0)\frac{\rho c_P\,\Delta P D^3}{12\mu L^2 q''}, \qquad \theta_2 = (T_2 - T_0)\frac{\rho c_P\,\Delta P D^3}{12\mu L^2 q''} \qquad (14)$$

The problem reduces to integrating for $\theta_1(\xi)$ and $\theta_2(\xi)$ the two equations

$$y^3\frac{d\theta_1}{d\xi} = 1 + B(\theta_2 - \theta_1) \qquad (15)$$

$$(1 - y)^3\frac{d\theta_2}{d\xi} = 1 - B(\theta_2 - \theta_1) \qquad (16)$$

by starting from the inlet, where $\theta_1(0) = \theta_2(0) = 0$. The dimensionless group B accounts for the transversal thermal conductance of the board,

$$B = 12\frac{k_w}{k}\frac{\mu\alpha L^2}{\Delta P \cdot D^3 t} \tag{17}$$

The solution for the temperature of the board surface facing the D_2 subchannel is

$$\theta_2(\xi) = \left(\frac{a}{p} - \frac{b}{p^2}\right)[1 - \exp(-p\xi)] + \frac{b}{p}\xi \tag{18}$$

with the shorthand notation

$$p = B\left[\frac{1}{(1-y)^3} + \frac{1}{y^3}\right], \quad a = \frac{1}{(1-y)^3}, \quad b = \frac{2B}{(1-y)^3 y^3} \tag{19}$$

The highest temperature occurs at the trailing edge, $\theta_{h,2} = \theta_2(1)$, namely,

$$\theta_{h,2} = \left(\frac{a}{p} - \frac{b}{p^2}\right)[1 - \exp(-p)] + \frac{b}{p} \tag{20}$$

This temperature is a function of the board position y and the board conductance parameter B, as shown by the solid curves in Figure 3. The highest temperature of the board surface facing the D_1 subchannel, $\theta_{h,1} = \theta_1(1)$, is obtained by switching y and $(1 - y)$ in the $\theta_{h,2}$ solution (20). The resulting family of curves $\theta_{h,1}(y, B)$ is superimposed with dash lines on Figure 3. On this composite graph, we seek the board position y_{min} that guarantees the lowest board temperatures when B is specified by design. The best board position y_{min} depends on B, i.e., on the degree to which the board substrate is a good thermal conductor:

(i) When B is of the order of 1 or larger, the $\theta_{h,1}$ and $\theta_{h,2}$ curves are bell shaped and fall on top of each other. The lowest temperatures are registered at $y_{min} = 0$ and $y_{min} = 1$, i.e., when the board is positioned close to one of the insulated walls of the channel. The worst position (called y_{max}) is in the middle of the channel, $y_{max} = \frac{1}{2}$, where the highest temperature rise ($\theta_{h,1}$, or $\theta_{h,2}$) is about four times greater than when the board is mounted close to one of the insulated walls. These conclusions agree all the way with what we learned in section 2 based on the isothermal board model (i.e., $B \to \infty$).

(ii) When the board is a poor thermal conductor, such that B is smaller than the order of 1, then $\theta_{h,1}$ and $\theta_{h,2}$ curves intersect forming a cusp right at $y = \frac{1}{2}$. That intersection corresponds to the lowest ($\theta_{h,1} = \theta_{h,2}$) values, indicating that the best position for the board is along the midplane of the $D \times L$ channel. The worst position, y_{max}, approaches 0.8 and 0.2 as B decreases.

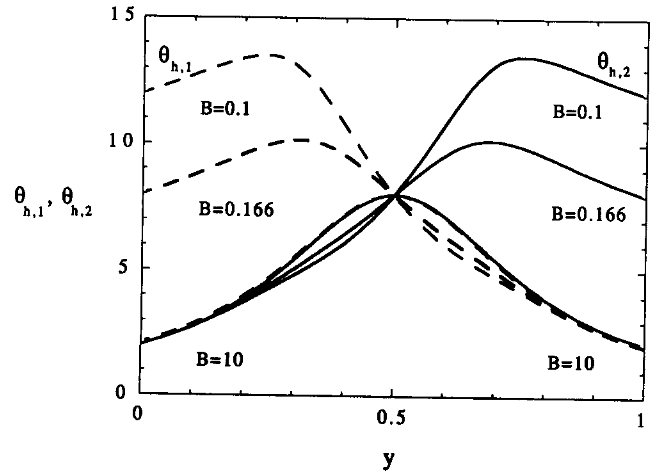

Figure 3 The trailing edge surface temperatures of a board with finite transversal thermal conductance B

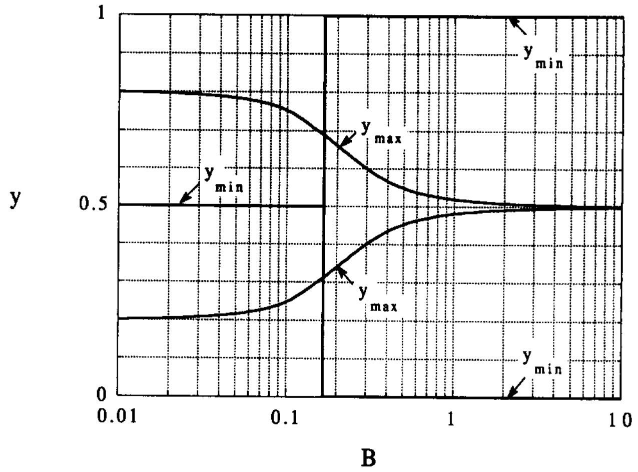

Figure 4 The best position (y_{min}) and the worst position (y_{max}) of a heat-generating board with finite transversal thermal conductance

These conclusions are summarized in Figure 4. The transition from conducting boards (i) to poorly conducting boards (ii) occurs when B drops below 0.166. This critical B is illustrated also in Figure 3.

It is fascinating that the best location for poorly conducting boards, $y_{min} = \frac{1}{2}$, happens to be exactly the same as the worst location for highly conducting boards. This observation stresses the crucial importance of the dimensionless number B. This number must be calculated early, in order to determine the problem type, (i) or (ii).

The lowest trailing-edge temperature ceiling that corresponds to the best location y_{min} is presented as θ_{min} versus B in Figure 5; in other words, $\theta_{min} = \min[\max(\theta_{h,1}, \theta_{h,2})]$. The same figure shows the uppermost trailing-edge temperature that corresponds to the worst position y_{max}, namely, $\theta_{max} = \max[\max(\theta_{h,1}, \theta_{h,2})]$. The lowest temperature ceiling (θ_{min}) is considerably smaller than the highest temperature ceiling (θ_{max}), regardless of the B value. This shows the importance of knowing not only the best design (y_{min}) but also the worst design (y_{max}) (see Bell 1992).

4. The optimal spacing of a channel with an isothermal heated plate attached to one of its walls

We now turn our attention to Figure 6, which reflects the conclusion drawn in the case of highly conducting boards in section 2. The heated plate is situated on or near one of the insulated walls of the channel. The design of a "package," or a stack of such cassettes, leads to the problem of determining the optimal channel spacing D for maximum conductance $q/(T_w - T_0)$, or minimum plate excess temperature $(T_w - T_0)$.

In this first treatment of the channel spacing problem, we continue to assume that the heated plate is isothermal at T_w (which is to be minimized), that the flow through the D wide channel is laminar and fully developed, and that all surfaces are smooth. We abandon assumption (c) of section 2, and recognize the more general relation

$$q = \dot{m}c_P(T_w - T_0)\left[1 - \exp\left(-\frac{hA_w}{\dot{m}c_P}\right)\right] \tag{21}$$

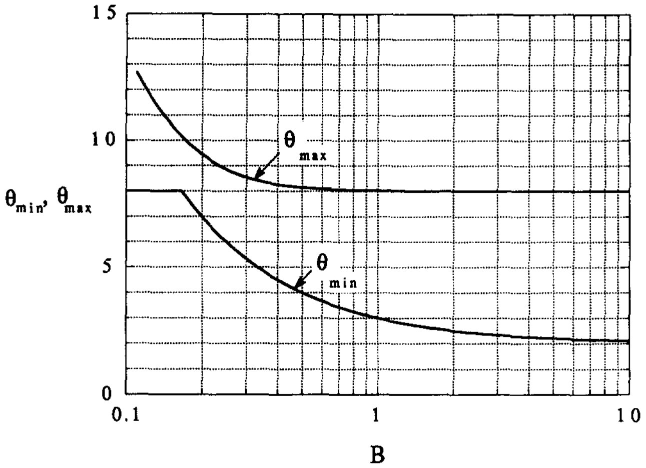

Figure 5 The lowest temperature ceiling (θ_{min}) and the uppermost temperature ceiling (θ_{max}) that correspond, respectively, to the best location (y_{min}) and the worst location (y_{max})

where

$$\dot{m} = \frac{\rho W D^3}{12\mu}\frac{\Delta P}{L} \tag{22}$$

The heat transfer coefficient is $h = \mathrm{Nu}\, k/D_h$, with $D_h = 2D$, and the Nusselt number Nu = 4.86 for fully developed flow and heat transfer in a channel with one side isothermal and the other side insulated (Kays and Crawford 1980). This h estimate, the heat transfer area $A_w = LW$, and Equation 22 can be used to rewrite the number of heat transfer units as

$$\mathrm{NTU} = \frac{hA_w}{\dot{m}c_P} = 6\mathrm{Nu}\left(\frac{L}{D}\right)^4 \frac{\mu\alpha}{\Delta P \cdot L^2} \tag{23}$$

On the right-hand side we see the emergence of an important group: the dimensionless imposed pressure difference,

$$\Pi = \frac{\Delta P \cdot L^2}{\mu\alpha} \tag{24}$$

The importance of nondimensionalizing ΔP in this way was noted earlier by Knight et al. (1991), who used $\Delta P{\cdot}L^2/\mu\nu$ instead of $\Delta P{\cdot}L^2/\mu\alpha$. Finally, Equation 21 becomes

$$\frac{q}{T_w - T_0}\,\frac{12}{Wk\Pi^{1/4}} = \delta^3[1 - \exp(-29.16\delta^{-4})] \tag{25}$$

where δ is the dimensionless channel spacing

$$\delta = \frac{D}{L}\,\Pi^{1/4} \tag{26}$$

The maximization of the right-hand side of Equation 25 with respect to δ reduces to solving the equation $\exp(\beta) = 1 + 4\beta/3$, with $\beta = 29.16\delta^{-4}$. The solution to this transcendental equation is $\beta = 0.5502$, which means that $\delta_{opt} = 2.70$, or that

$$\frac{D_{opt}}{L} = 2.70\left(\frac{\mu\alpha}{\Delta P \cdot L^2}\right)^{1/4} \tag{27}$$

By substituting Equation 27 into Equation 25, we obtain the maximum thermal conductance,

$$\left(\frac{q}{T_w - T_0}\right)_{max} = 0.693Wk\left(\frac{\Delta P \cdot L^2}{\mu\alpha}\right)^{1/4} \tag{28}$$

or, if we write $\bar{q}'' = q/WL$ for the average heat flux,

$$\left(\frac{\bar{q}''}{T_w - T_0}\right)_{max}\frac{L}{k} = 0.693\left(\frac{\Delta P \cdot L^2}{\mu\alpha}\right)^{1/4} \tag{29}$$

In conclusion, when the pressure difference ΔP is held fixed, the optimal spacing varies as $L^{1/2}$, and the maximum conductance varies as $L^{-1/2}$.

5. The optimal spacing of a channel with uniform-flux board

In this section we reconsider the problem defined in the preceding section, by modeling the circuit board as a plate with uniform heat flux, q''. Such a plate reaches its highest temperature (T_h) at the trailing edge, i.e., in the plane of the outlet (Figure 7). This model is more appropriate for boards that are relatively poor thermal conductors (small B, section 3), which are the coldest when placed in the middle of the channel of Figure 1. This means that in Figure 7, the D channel would correspond to $D/2$ in Figure 1, or that the bottom insulated side of Figure 7 represents the plane of symmetry of the board mounted in the middle of the channel of Figure 1.

The objective is to find the optimal spacing D that minimizes T_h, or maximizes the overall thermal conductance expressed as the ratio $q''L/(T_h - T_0)$. We continue to assume that the flow and heat transfer are laminar and fully developed, and that the surfaces are smooth. The relation between the highest wall temperature (T_h) and the outlet temperature of the stream (T_{out}) is

$$T_h - T_{out} = \frac{q''2D}{k\mathrm{Nu}} \tag{30}$$

where Nu = 5.385 for a channel with one uniform-flux surface, and the other surface insulated (Kays and Crawford 1980). The stream outlet temperature can be calculated by invoking the first law of thermodynamics for the entire stream,

$$T_{out} - T_0 = \frac{q''L}{\rho c_P U D} \tag{31}$$

where U is the mean velocity associated with ΔP and D,

$$U = \frac{\Delta P \cdot D^2}{12\mu L} \tag{32}$$

Next, we eliminate T_{out} between Equations 30 and 31, and

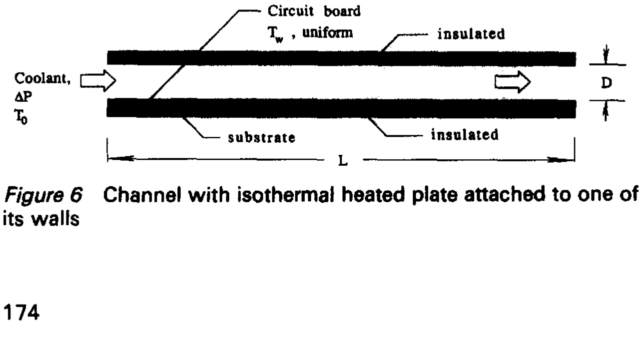

Figure 6 Channel with isothermal heated plate attached to one of its walls

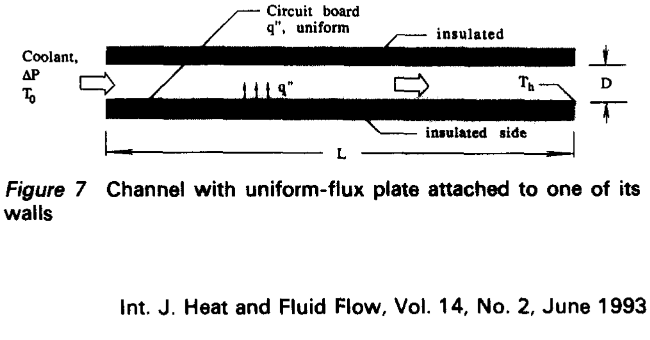

Figure 7 Channel with uniform-flux plate attached to one of its walls

arrange the overall thermal resistance in the following nondimensional way:

$$\frac{T_h - T_0}{q''L/k} = \frac{12}{\Pi}\left(\frac{L}{D}\right)^3 + \frac{2}{\mathrm{Nu}}\frac{D}{L} \tag{33}$$

This quantity (or T_h) can be minimized analytically with respect to the spacing D, and the result is

$$\frac{D_{opt}}{L} = 3.14\left(\frac{\mu\alpha}{\Delta P \cdot L^2}\right)^{1/4} \tag{34}$$

The corresponding maximum thermal conductance is obtained by combining Equations 33 and 34:

$$\left(\frac{q''}{T_h - T_0}\right)_{max}\frac{L}{k} = 0.644\left(\frac{\Delta P \cdot L^2}{\mu\alpha}\right)^{1/4} \tag{35}$$

The similarities between the uniform-flux results (Equations 34 and 35) and the corresponding uniform-temperature results (Equations 27 and 29) are evident. The optimal spacing of the channel with a uniform-flux circuit board (Equation 34) is only 16 percent greater than the optimal spacing calculated based on the isothermal plate model (Equation 27). This comparison holds true when the ceiling temperature of the board is the same in both models, $T_h = T_w$.

Equations 29 and 35 show that the maximum thermal conductance of the isothermal board exceeds by only 8 percent the maximum thermal conductance of the board with uniform heat flux. Responsible for this difference is the leading section of the isothermal board, which is considerably warmer (higher above T_0) than the leading section of the uniform-flux board.

In conclusion, the choice of a thermal boundary condition for the exposed surface of the heated plate has little effect on the design estimates for optimal channel spacing and maximum thermal conductance.

6. Should the single board be replaced with two or more equidistant boards?

Consider now a somewhat different question regarding the optimal cooling of a heat-generating board inside a channel. As shown in Figure 8, the channel has the spacing D and length L, while the board thickness is negligible. The amount of electronics, or the total rate of heat transfer released through both sides of the board, q, is fixed. Each side is a smooth uniform-flux surface. The objective continues to be minimization of the board's highest temperature.

The new question is this: Should we optimize the position occupied by the single board in the channel, as done in sections 2 and 3, or should we distribute the given electronics (q) equally on two parallel boards? Or, is it better to use three parallel boards, while $q/3$ is being generated by each board?

If the original board is replaced with n equally loaded boards, then each board releases q/n into the surrounding fluid. If the boards are spaced equidistantly, the spacing of each subchannel is

$$D_n = \frac{D}{n+1} \tag{36}$$

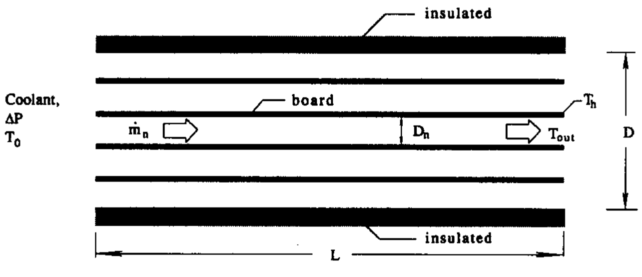

Figure 8 Stack of n equidistant boards, replacing the single board in the parallel-plate channel of Figure 1

The heat transfer rate received by the stream that flows through one subchannel is q/n. For the sake of simplicity, we assume that the extreme (first and last) subchannels are identical to the internal ones. This approximation becomes better as n increases. It is a conservative approximation, because the true heat transfer rate to an extreme subchannel is $(q/n)/2$, i.e., only half of the heat transfer rate to one of the internal subchannels. This means that the highest temperature occurs on the internal boards. That temperature, T_h, is located in the plane of the outlet, where the bulk temperature of the subchannel stream is T_{out}.

Assuming that the subchannel flow is laminar and fully developed, the Nusselt number for the parallel-plate channel with uniform flux on both surfaces (Kays and Crawford 1980) is Nu = 8.235, where Nu is based on the hydraulic diameter, $\mathrm{Nu} = h2D_n/k$, and, in the plane of the outlet, $h = q_n''/(T_h - T_{out})$. The uniform heat flux on each subchannel is $q_n'' = (q/n)/(2LW)$. In conclusion, the relation between the highest temperature and the outlet temperature is

$$T_h - T_{out} = \frac{q}{k\mathrm{Nu}LW}\frac{D}{n(n+1)} \tag{37}$$

The relation between the stream outlet temperature and the inlet temperature T_0 follows from the first law of thermodynamics for one subchannel:

$$T_{out} - T_0 = \frac{n+1}{n}\frac{q}{\rho c_P \mathrm{UDW}} \tag{38}$$

The average velocity through the subchannel, U, is obtained quite easily by rewriting Equation 22, i.e., with D_n in place of D, and $\dot{m}_n = \rho U D_n W$ in place of $\dot{m}$. Finally, Equations 37 and 38 can be added side by side to construct the following expression for the temperature ceiling (highest excess temperature) as a function of the number of boards:

$$\frac{T_h - T_0}{q/Wk}\cdot\Pi^{1/4} = \frac{1}{n(n+1)\mathrm{Nu}}\left(\frac{D}{L}\Pi^{1/4}\right) + 12\frac{(n+1)^3}{n}\left(\frac{D}{L}\Pi^{1/4}\right)^{-3} \tag{39}$$

The overall pressure drop number Π was defined in Equation 24. The group $(D/L)\Pi^{1/4}$, which appears on the right-hand side of Equation 39, is the important "external" parameter of the overall space ($L \times D \times W$) in which the division of the original board is being contemplated.

Figure 9 shows that the number of boards n that minimizes the highest temperature depends on the value of the external parameter $(D/L)\Pi^{1/4}$. The best n increases as the abscissa parameter $(D/L)\Pi^{1/4}$ increases. If the group $(D/L)\Pi^{1/4}$ is greater than the order of 10, the recommended number of boards is large so that $(n+1) \cong n$. In this limit, the right-hand side of Equation 39 can be minimized analytically with respect to n to obtain the number of boards for minimum $(T_h - T_0)$ at fixed Π and $(D/L)\Pi^{1/4}$,

$$n_{opt} \cong \frac{1}{3.15}\frac{D}{L}\Pi^{1/4} \tag{40}$$

Substituted into Equation 39, the number of boards n_{opt} leads

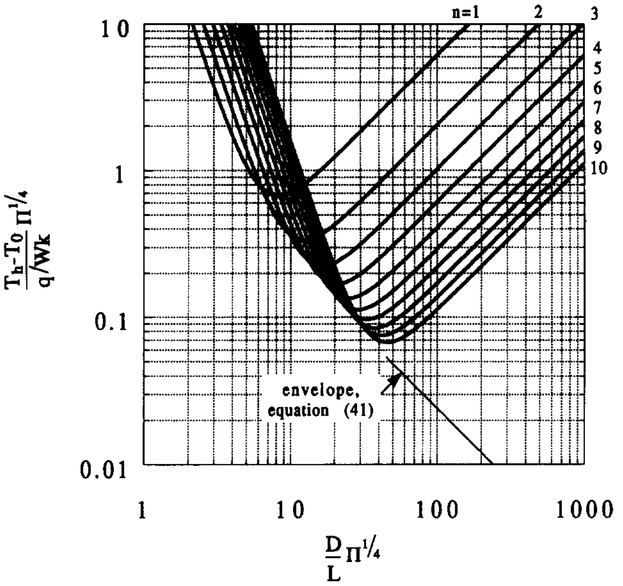

Figure 9 The effect of the number of boards on the highest temperature at the board trailing edge

to the corresponding minimum temperature ceiling:

$$\left(\frac{T_h - T_0}{q/Wk}\right)_{\min} = 2.41 \frac{L}{D} \Pi^{-1/2} \tag{41}$$

Rounded off to the closest integer, the n_{opt} value calculated based on Equation 40 reproduces quite well the n value that would be needed in order to place the design on the envelope of the family of n = constant curves plotted in Figure 9. Recall that $(D/L)\Pi^{1/4}$ is given. Note further that if we set $n_{\text{opt}} = 1$ in Equation 40, we reproduce fairly well Equations 27 and 34, which should have been expected.

The lowest temperature ceiling that is associated with operating on the envelope (Equation 41), $[(T_h - T_0)/(q/Wk)]_{\min}$, decreases as $1/n$ as n increases. The returns from installing an additional board diminish significantly as n becomes greater than the order of 10.

It is important to keep in mind that the present assumption that the flow channels are identical is a major simplification that may not correspond to actual applications. It is well known that the flow through multiple channels tends to be distributed nonuniformly (Rohsenow et al. 1985; Shah and London 1978), due to the different distributions of electronics on the various channel surfaces. A useful extension of the analysis presented in this section would be to consider multiple channels that are not identical, and to employ the method developed by Shah and London (1978).

7. Conclusions

In this paper, we have addressed the fundamental thermal design question of how to position a heat-generating board that is inserted in a parallel-plate channel. The main conclusions of this work are as follows.

1. When the board is a good enough thermal conductor such that the B parameter defined in Equation 17 is greater than 0.166, optimal cooling occurs when the board is positioned near one of the walls of the channel. In this range of B values, the worst board position is in the middle of the parallel-plate channel (Figure 4).
2. When the board is a relatively poor thermal conductor in the transversal direction, $B < 0.166$, the board position for maximum cooling (minimum temperature) is in the middle of the channel. The worst position is near one of the walls of the channel (Figure 4).
3. The optimal spacing between a heat-generating surface with uniform temperature and the insulated wall that completes a parallel-plate channel is given by Equation 27. The corresponding maximum thermal conductance is reported in Equation 29.
4. The optimal spacing between a heat-generating surface with uniform heat flux and the insulated wall of the parallel-plate channel is given by Equation 34. The minimum thermal conductance is given by Equation 35.
5. Figure 9 and Equations 40 and 41 answer the question of whether a heat-generating board should be divided into two or more equidistant boards inside the same channel with fixed spacing D. The overall rate of heat generation of the n boards is fixed, q. The optimal number of boards scales as $(D/L)\Pi^{1/4}$, while the minimum temperature scales as $(L/D)\Pi^{-1/2}$.

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